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Maths Built the Grid – Can Maths Revitalise the Grid?

How Did Maths Build the Grid?

> *'The grid is made up of millions of individual pieces of equipment, all bought at the lowest tender price.'* *-Anon*

Fortunately, each of those millions of individual pieces has electrical characteristics that can be:
- Calculated and/or
- Measured

Consequently, the performance of the grid, as a complete system, can be computed to a relatively high degree of accuracy.

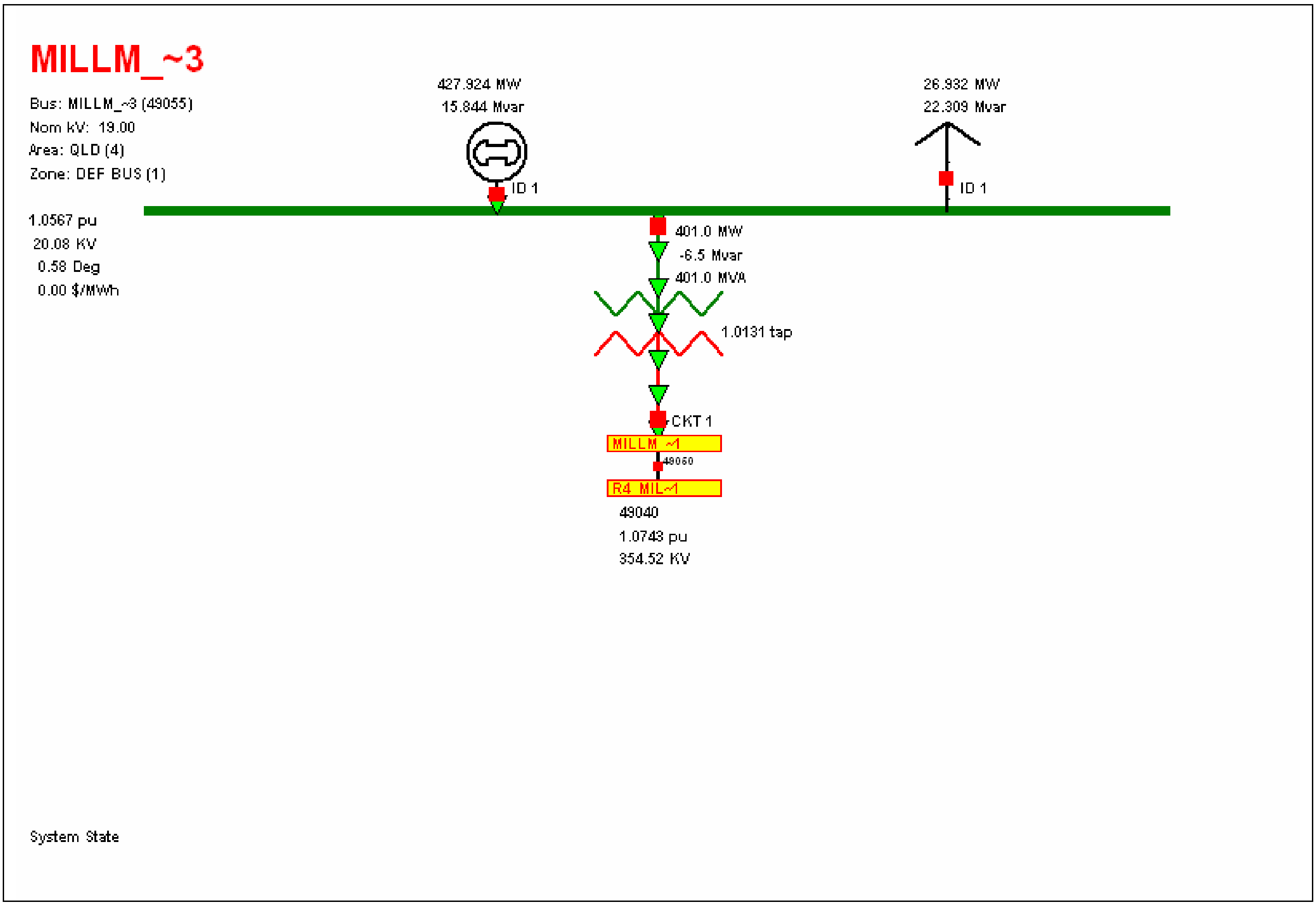
By any standards, the computation of grid models occurs on a large scale. This is feasible due to major advances, firstly in mathematical modelling and software development, and more recently in computer hardware.

For instance, commercially available software can solve power flows up to 100,000 buses, or sufficient for the entire US grid east of the Mississippi.

It can do this down to a level of accuracy (or mismatch) of no more than 0.1MW at any bus (or node). That requires a resolution of 0.1MW in 500,000MW of peak demand.

The solution to the power flow requires that Kirchhoff's Law (currents injected at a node, must always sum to zero) be observed. The AC power flow problem is nonlinear, since it requires that nett power injection at each node (the power generated minus the power demand) is a fixed value. Since voltages across the grid are not uniform (they are unknowns), the AC power flow must therefore be formulated as a nonlinear set of equations.

The following picture shows the solution to the power flow at one node in the Australian National Electricity Market (NEM) out of several thousand.



In mathematical terms, at each node, in polar coordinates:

$$\Delta Pi = Pi^{sp} - Vi\sum_{k=1}^{n} Vk\left(GikCos\theta ik + BikSin\theta ik\right)$$

$$\Delta Qi = Qi^{sp} - Vi\sum_{k=1}^{n} Vk\left(GikSin\theta ik - BikCos\theta ik\right)$$

Where: ΔP and ΔQi are the real and reactive power mismatch at node *i*
Pi^{sp} and Qi^{sp} are the nett power injection at node *i*
$Vi, \theta i$ are the voltage magnitude and angle at node *i*
$Vk, \theta k$ are the voltage magnitude and angle at node(s) *k*
Gik and Bik are the real and reactive impedances of line *ik*

The power flow is solved by making an initial guess at the voltages throughout the grid (usually a ‘flat start’ with all voltage magnitudes set to unity and all angles set to zero).

The computation then proceeds by firstly computing the initial estimate of angles θ from the first equation (this is known as the solution to the 'DC load flow').

$$\left[\frac{\Delta P}{V}\right] = [H][\Delta\theta]$$

Where $Hik = Vk(GikSin\theta ik - BikCos\theta ik)$

However, since θik is normally less than 30°:

$$Cos\theta ik \rangle Sin\theta ik$$

$$Bik \rangle Gik$$

$$\therefore BikCos\theta ik \rangle\rangle GikSin\theta ik$$

and $BikCos\theta ik \approx Bik$

and $Vk \approx 1$

$$\therefore \left[\frac{\Delta P}{V}\right] = [Bik][\Delta\theta]$$

and $\left[\frac{\Delta Q}{V}\right] = [Bik][\Delta V]$

The DC load flow computes $\Delta\theta$'s from the initial ΔP's. Then the flows are calculated from the calculated angular differences between the nodes in the network.

The full solution to the AC power flow relies on a number of mathematical innovations to avoiding inverting a matrix of up to 100,000 × 100,000 or 10 billion terms to obtain the solution:

i. Ordered elimination of nodes to conserve the sparsity of the matrix of connection between nodes (because each node is normally connected only to a few adjacent nodes)
ii. Separation of the real and reactive terms of the power flow by alternately computing $\Delta\theta$ and ΔV, thus halving the size of the matrix to be solved
iii. Approximating the matrix terms to eliminate trigonometric functions without affecting the final solution (since only the rate of convergence to the solution is altered)
iv. Forward and backward substitution to obtain the solution in a computationally efficient manner

This mathematical solution is now able to be obtained for very large networks in a matter of seconds and is thus applicable for use in real time in control centres such as the NEM dispatch centres. The AC power flow accurately computes the voltages at all nodes and the resulting real and reactive power flows on all lines for a known or measured set of power demands and generation inputs. The operators are therefore presented with an accurate picture of the system state including any transmission lines that are loaded beyond their normal ratings. However the AC power flow does not provide guidance to the operator as to the economic means of removing overloads through re-dispatching generation.

The concept of constrained economic dispatch therefore resulted from a real requirement to limit the possibility of line overloading by re-dispatching, or altering in real time the pattern of generation output in the grid. This concept predated the emergence of competitive electricity markets by many years.

Linear Programming (LP) took over from Lagrangian relaxation in the 1980's in the US, Australia and elsewhere and has emerged as the preferred mathematical solution to generation dispatch owing to its computational feasibility, robustness, and capability to reasonably accurately depict the real network behaviour.

The LP formulation is:

Minimise $F = C^t \cdot P$

Where F is production cost in dollars per hour

> C^t is the transposed vector of incremental generation cost ($/MWh) and is the product of the slope of the heat rate curve, the fuel cost and the transmission loss penalty factor for each generator (the marginal loss factors defined for the NEM are the inverse of the traditional penalty factor)
>
> P is the vector of gross generation

Subject to:

i. Power balance equality constraint
ii. Maximum and minimum generator capacity limits
iii. Minimum reserve limits
iv. Maximum and minimum interconnector limits
v. DC power flow constraint equations (if implemented)

For a 'hub and spoke' or 'radial' connection of major lines, the flows on interconnectors may be computed without reference to the power flow solution and therefore do not require a DC load flow network model.

This is the model presently used for the NEM with costs replaced by bid offers to supply energy. The NEM is represented as a linear connection of six regions containing generator/load centres. As such the flows on the interconnectors can be represented in the LP algorithms either as flow terms or as functions of generator output.

Limitations in Market Modelling

The entire objective of the electricity market is to enable the market to identify the optimum least cost development of the generation and transmission system by providing a mechanism for signalling through nodal prices the existence or emergence of generation and load imbalances relative to the transmission grid capacity in a timely fashion. It is then up to the market to develop generation or demand side capacity, with transmission network service providers (TNSPs) augmenting the grid when the market would not bring forth a solution.

When we consider that objective in relation to electricity market developments, we see that a variety of approximations may be impeding the effective functioning of the market:

1) AC power flow equations have been replaced by DC equations for most market modelling, including the New Zealand market (the Australian market does not include meshed network models or modelling of individual lines forming an interconnector, even though this would be an advancement).
2) Losses between regions, which vary with the square of the current (or power) transfers, are represented by a piecewise linear methodology, since the LP methodology cannot incorporate nonlinear terms.
3) Transmission limit equations (the Right Hand Side of Constraint Equations) are linearised representations following hundreds or thousands of off-line calculations made by TNSPs, from which linear regression equations are formulated for application in dispatch.
4) Constraint equations in LP are linearised representations of the factors that relate changes in each generator's output to the resulting change in flows on critical transmission lines.
5) The NEM dispatch rules require that transmission line flows do not exceed limits even following the worst single contingency (line or generator) outage event. Thus limit equations are formulated off-line for a wide range of specified contingencies followed by selection of those that are judged more critical for inclusion in the constraint equation set. These could, in principle, be replaced by an on-line security constrained constraint set and incorporated directly into the LP dispatch methodology.
6) Static marginal intra-regional loss factors[1] are computed annually off-line and used as an approximation to dynamic marginal loss factors (inter-regional loss factors are computed on-line and used in 5-minute dispatch however). All marginal loss factors could be computed on-line instead on a 5-minute or half-hourly basis.

In summary, we have a situation where, while the AC power flow[2] provides a rigorous calculation of the present system state and creates the means for accurate nodal dispatch and pricing, the existing set of market nodal or

[1] Marginal Loss Factors are used within dispatch to adjust the relative bids in favour of the generators that cause lower transmission losses.
[2] The Least Squares State Estimation technique is used in conjunction with network equations to create a base case of data to conduct power flow and contingency calculations.

regional pricing equations devised from the load flow principles rely on a set of approximations that may be well removed from the theoretical source.

This introduces many distortions, such that considerable effort is expended by market participants in understanding the flaws in the models, to enable advantage to be taken where possible.

Have we now gone too far in 'dumbing down' the modelling to allow for simplified commercial transactions?

Have we also created a situation where market participants no longer have reliable information on which to make long term investment decisions?

Should we now be moving towards improved on line solutions?

This could include:

- On-line limit calculations, including stability and thermal
- On-line formulation of constraint equations
- On-line full AC power flow nodal dispatch
- On-line MLF calculation
- On-line emissions constrained dispatch

Should we now also be relying on state-of-the-art computer models to inform the market that are capable of simultaneously:

- Trading off generation, demand side and transmission investments over the lifetime of these assets;
- Incorporating various emission constraints such as emissions caps and carbon taxes; and
- Assessing the least cost means of providing reliable, secure electricity supplies?

Such models would provide clear guidance to the market as to the benefits of choosing appropriate technologies and locations for new generation, transmission and demand side equipment.

If we do not do this, we risk a situation where the original objectives of the formation of the market, listed below, are no longer achieved:

- The market should be competitive;
- Customers should be able to choose which supplier (including generators and retailers) they will trade with;
- Any person wishing to do so should be able to gain access to the interconnected transmission and distribution network;
- A person wishing to enter the market should not be treated more favourably or less favourably than if that person were already participating in the market;
- A particular energy source or technology should not be treated more favourably or less favourably than another energy source or technology; and

- The provisions regulating trading of electricity in the market should not treat intrastate trading more favourably or less favourably than interstate trading of electricity.

At present, we risk a proliferation of rules that are at best 'fudge factors' to deal with the inability of present models to accurately reflect the behaviour of the grid.

Future Directions

Arguably, the most effective starting point is to improve long range planning of the grid, including serious consideration of upgrading network capacity at critical locations. Such upgrades would have the multiple benefits of improving generation competitiveness, lowering transmission losses, improving security and reliability of supply and alleviating congestion.

This approach has been recommended by the Energy Reform Implementation Group (ERIG) in its April 2007 report, and endorsed by the Council of Australian Governments.

The ERIG has determined that nodal modelling is not the answer, owing to several risks that could emerge, including the ability on the part of some participants to exert market power. However, ERIG does favour a closer examination of the deficiencies of the present models, to deliver some of the benefits of locational pricing.

The next stage of market development in Australia opens the way for comprehensive input by mathematicians, engineers, economists and other technical specialists to deliver the techniques for advanced modelling of the grid.

It will be up to the mathematicians and technical leaders to deliver the new techniques to administrators and bureaucrats, often before they are called upon, in order to deliver a safe, reliable, secure, and environmentally responsible power system for the 21st century.