Fields Medal
Terence Tao

CITATION:
"For his contributions to partial differential equations, combinatorics, harmonic analysis and additive number theory"

Terence Tao is a supreme problem-solver whose spectacular work has had an impact across several mathematical areas. He combines sheer technical power, an other-worldly ingenuity for hitting upon new ideas, and a startlingly natural point of view that leaves other mathematicians wondering, "Why didn't anyone see that before?"

At 31 years of age, Tao has written over 80 research papers, with over 30 collaborators, and his interests range over a wide swath of mathematics, including harmonic analysis, nonlinear partial differential equations, and combinatorics. "I work in a number of areas, but I don't view them as being disconnected," he said in an interview published in the Clay Mathematics Institute Annual Report. "I tend to view mathematics as a unified subject and am particularly happy when I get the opportunity to work on a project that involves several fields at once."

Because of the wide range of his accomplishments, it is difficult to give a brief summary of Tao's oeuvre. A few highlights can give an inkling of the breadth and depth of the work of this extraordinary mathematician.

The first highlight is Tao's work with Ben Green, a dramatic new result about the fundamental building blocks of mathematics, the prime numbers. Green and Tao tackled a classical question that was probably first asked a couple of centuries ago: Does the set of prime numbers contain arithmetic progressions of any length? An "arithmetic progression" is a sequence of whole numbers that differ by a fixed amount: 3, 5, 7 is an arithmetic progression of length 3, where the numbers differ by 2; 109, 219, 329, 439, 549 is a progression of length 5, where the numbers differ by 110. A big advance in understanding arithmetic progressions came in 1974, when the Hungarian mathematician Emre Szemeredi proved that any infinite set of numbers that has "positive density" contains arithmetic progressions of any length. A set has positive density if, for a sufficiently large number n, there is always a fixed percentage of elements of {1, 2, 3, ... n} in the set. Szemeredi's theorem can be seen from different points of view, and there are now at least three different proofs of it, including Szemeredi's original proof and one by 1998 Fields Medalist Timothy Gowers. The primes do not have positive density, so Szemeredi's theorem does not apply to them; in fact, the primes get sparser and sparser as the integers stretch out towards infinity. Remarkably, Green and Tao proved that, despite this sparseness, the primes do contain arithmetic progressions of any length. Any result that sheds new light on properties of prime numbers marks a significant advance. This work shows great originality and insight and provides a solution to a deep, fundamental, and difficult problem.

Another highlight of Tao's research is his work on the Kakeya Problem, which in its original form can be described in the following way. Suppose you have a needle lying flat on a plane. Imagine the different possible shapes swept out when you rotate the needle 180 degrees. One possible shape is a half-disk; with a bit more care, you can perform the rotation within a quarter-disk. The Kakeya problem asks, What is the minimum area of the shape swept out in rotating the needle 180 degrees? The surprising answer is that the area can be made as small as you like, so in some sense the minimum area is zero.
The fractal dimension of the shape swept out provides a finer kind of information about the size of the shape than you obtain in measuring its area. A fundamental result about the Kakeya problem says that the fractal dimension of the shape swept out by the needle is always 2.

Imagine now that the needle is not in a flat plane, but in n-dimensional space, where n is bigger than 2. The n-dimensional Kakeya problem asks, What is the minimum volume of an n-dimensional shape in which the needle can be turned in any direction? Analogously with the 2-dimensional case, this volume can be made as small as you like. But a more crucial question is, What can be said about the fractal dimension of this n-dimensional shape? No one knows the answer to that question. The technique of the proof that, in the 2-dimensional plane the fractal dimension is always 2, does not work in higher dimensions. The n-dimensional Kakeya problem is interesting in its own right and also has fundamental connections to other problems in mathematics in, for example, Fourier analysis and nonlinear waves. Terence Tao has been a major force in recent years in investigating the Kakeya problem in n dimensions and in elucidating its connections to other problems in the field.

Another problem Tao has worked on is understanding wave maps. This topic arises naturally in the study of Einstein's theory of general relativity, according to which gravity is a nonlinear wave. No one knows how to solve completely the equations of general relativity that describe gravity; they are simply beyond current understanding. However, the equations become far simpler if one considers a special case, in which the equations have cylindrical symmetry. One aspect of this simpler case is called the "wave maps" problem, and Tao has developed a program that would allow one to understand its solution.
While this work has not reached a definitive endpoint, Tao's ideas have removed a major psychological obstacle by demonstrating that the equations are not intractable, thereby causing a resurgence of interest in this problem.

A fourth highlight of Tao's work centers on the nonlinear Schroedinger equations. One use of these equations is to describe the behavior of light in a fiber optic cable. Tao's work has brought new insights into the behavior of one particular Schroedinger equation and has produced definitive existence results for solutions. He did this work in collaboration with four other mathematicians, James Colliander, Markus Keel, Gigliola Staffilani, and Hideo Takaoka. Together they have become known as the "I-team", where "I" denotes many different things, including "interaction". The word refers to the way that light can interact with itself in a medium such as a fiber optic cable; this self-interaction is reflected in the nonlinear term in the Schroedinger equation that the team studied. The word "interaction" also refers to interactions among the team members, and indeed collaboration is a hallmark of Tao's work. "Collaboration is very important for me, as it allows me to learn about other fields, and, conversely, to share what I have learnt about my own fields with others," he said in the Clay Institute interview. "It broadens my experience, not just in a technical mathematical sense, but also in being exposed to other philosophies of research and exposition."

These highlights of Tao's work do not tell the whole story. For example, many mathematicians were startled when Tao and co-author Allen Knutson produced beautiful work on a problem known as Horn's conjecture, which arises in an area that one would expect to be very far from Tao's expertise. This is akin to a leading English-language novelist suddenly producing the definitive Russian novel. Tao's versatility, depth, and technical prowess ensure that he will remain a powerful force in mathematics in the decades to come.

Terence Tao was born in Adelaide, Australia, in 1975. He received his PhD in mathematics in 1996 from Princeton University. He is a professor of mathematics at the University of California, Los Angeles. Among his distinctions are a Sloan Foundation Fellowship, a Packard Foundation Fellowship, and a Clay Mathematics Institute Prize Fellowship. He was awarded the Salem Prize (2000), the American Mathematical Society (AMS) Bocher Prize (2002), and the AMS Conant Prize (2005, jointly with Allen Knutson).

Tubes that are transverse can have smaller intersection, and thus larger union, than tubes that are nearly parallel. Recent progress on problems such as the Kakeya conjecture has been aided by a "bilinear" approach that excludes the latter case from consideration.