Mathematical Optimization of Water Distribution Systems using Mixed Integer Nonlinear Programming (MINLP)

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TOPICS

- Water Distribution Systems
- Mathematical Optimization
- MINLP solvers
- Performance of different MINLP solvers
- High Performance Computing in Mathematical Optimization
- Conclusion
- Discussion

Water Distribution Systems
Problem Definition

- Find the near optimal solution of the diameter of each pipe
- Design parameters are fixed, for example, the length of the pipe and the topology of the water systems
- Capture the main design properties of an actual application of a city in Indonesia
- Solved by Mixed Integer Nonlinear Programming (MINLP)

Mathematical Optimization Terms

- Decision variables
- Objective function
- Constraints

Mathematical Optimization Steps

- Decision process
- Identification process and clarification
- Problem definition
- Development of a mathematical model and measure of effectiveness
- Deriving a solution
- Sensitivity analysis
- Testing the solution
- Implementation

Mathematical Optimization Modeling Languages

There are two major modeling languages

- AMPL A Mathematical Programming Language
- GAMS General Algebraic Modeling System

How it works

- Input : program written in mathematical modeling language
- Process :

1. Modeler translates the program into a form that can be read by solver
2. Solver solves the program
3. Solver sends the result to Modeler

- Output : Modeler displays the result to user

Mathematical Optimization
Different Types of Problems

- Bound Constrained Optimization
- Combinatorial Optimization
- Complementary Problems
- Global Optimization
- Linear Network Programming
- Linear Programming
- Mixed Integer Linear Programming
- Nonlinearly Constrained Optimization
- Non-differentiable Optimization
- Semi-definite Programming
- Semi-infinite Optimization
- Stochastic Linear Programming
- Second Order Conic Programming
- Unconstrained Optimization
- Mixed Integer Nonlinearly Constrained Optimization (MINLP)

Mathematical Optimization
Characteristics of MINLP

- Binary variables and/or
- Integer variables

Please note that Binary variables can be represented by Integer variables just like $x>=0$ and $x<=1$

Mathematical Optimization
AMPL/GAMS MINLP solvers

AMPL MINLP solvers

- BONMIN : NLP-based branch-and-boundn, outer-approximation decomposition and branch-and-cut
- CPLEX : large-scale mixed-integer programs from ILOG, INC.
- FILMINT : MINTO branch-and-cut framework for MILP with FILTERSQP
- LAGO : Lagrangian Global Optimizer for nonconvex mixed-integer nonlinear programs
- MINLP : Branch-and-Bound algorithm from R. Fletcher and S. Leyffer, The University of Dundee
- KNITRO : General purpose Large scale nonlinear optimization software from Ziena company

GAMS MINLP solvers

- ALPHAECP : Extended Cutting Plane Algorithm from T.Westerlund, Abo Akademi University, Finland
- BARON : Branch-and-Reduce algorithm from N. Sahinidis, University of Illinois Urbana-Champaign
- DICOPT : Outer-Approximation algorithm from I.E. Grossmann, Carnegie Mellon University
- LOGMIP : LogMIP (acronym of Logical Mixed Integer Programming) is a solver for or generalized disjunctive programs
- LINDOGLOBAL : LindoGlobal is a solver for proven global solutions from Lindo Systems, Inc.
- MINLP : Branch-and-Bound algorithm from R. Fletcher and S. Leyffer, The University of Dundee
- SBB : Branch-and-Bound algorithm from ARKI Consulting and Development

NOTE : Baron, Cplex, Dicopt, Knitro and Lindoglobal are commercial products

Water Distribution Systems GAMS program listing

```
:
Positive variables qp, qn(n,np)
Binary variable qb(n,np);
Equations               cont(n)       flow conservation equation at each node
                        loss(n,n)     pressure loss on each arc
                        peq           pump cost equation
                        deq           investment cost equation
                        weq           water cost equation
                        obj           objective function
                        dpen          penalty definition
                        qpup(n,np)    positive bounds
                        qnup(n,np)    negative bounds ;
:
qpup(a)..   qp(a) =l=  maxq*qb(a);
qnup(a)..   qn(a) =l=  maxq*(1-qb(a));
:
```

Note : Binary variable qb(n,np)

Water Distribution Systems result from SBB solver

```
---- VAR qb

           LOWER UPPER
           LEVEL   LEVEL   MARGINAL

nw.cc          1.000     1.000     EPS
nw.w           1.000     1.000     EPS
nw.n                          0.000     1.000  -226.242
e .cc                         1.000     1.000     4.607
e .s           1.000     1.000     EPS
e .se                         1.000     1.000     EPS
e .n           1.000     1.000     EPS
cc.W                          0.000     1.000   -14.449
cc.Sw          0.000     1.000  -200.498
cc.S           1.000     1.000     EPS
cc.N                          0.000     1.000  -136.113
sw.w                          0.000     1.000     .
s .sw          0.000     1.000   -42.829
s .se                         0.000     1.000     EPS
```

Water Distribution Systems
recent results from papers

- "Global Optimization of Water Distribution Systems" by Kaj-Mikael Bjork, Lazaros G. Papageorgiou, Proceedings of the Proceedings of the 41st Annual Hawaii International Conference on System Sciences, 2008.
- "Water Network Design by MINLP" by Cristiana Bragalli, Claudia D'Ambrosio, Jon Lee, Andrea Lodi, Paolo Toth, IBM Research Report RC24495, February 2008.
- "Optimal Design of Hybrid Membrane Networks for Wastewater Treatment" by Yousef Saif, University of Waterloo, Canada, PhD Thesis, May 2008.
- "Design Energy Efficient Water Utilization Systems Allowing Operation Split", LIAO Zuwei, WU Jintao, JIANG Binbo, WANG Jingdai and YANG Yongrong, Chinese Journal of Chemical Engineering, 16(1) p.16—20, 2008.
- "Optimisation of design and operation of MSF desalination process using MINLP technique in gPROMS" by M.S. Tanvir, I.M. Mujtaba (UK), p.419-430, Desalination, Vol. 222 (2008)

MINLP Solvers

Performance : Sample program

```
var x0 integer >= 1 <= 10;
var y0 integer >= 1 <= 10;
var x1 >= 1 <= 10 ;
var y1 >= 1 <= 10;
var x2 >= 1 <= 10 ;
var y2 >= 1 <= 10 ;
maximize dist:  (x0-x1)**2 + (y0-y1)**2 + (x0-x2)**2 +
                (y0-y2)**2 + (x1-x2)**2 + (y1-y2)**2 ;
subject to
e01: (x0-x1)**2 + (y0-y1)**2 >= 12.5 ;
e10: (x0-x1)**2 + (y0-y1)**2 <= 64 ;
e02: (x0-x2)**2 + (y0-y2)**2 >= 12.5 ;
e20: (x0-x2)**2 + (y0-y2)**2 <= 64 ;
e12: (x1-x2)**2 + (y1-y2)**2 >= 12.5 ;
e21: (x1-x2)**2 + (y1-y2)**2 <= 64 ;
solve ;
display x0,y0,x1, y1 ,x2, y2  ;
```

MINLP Solvers

Performance : Results

- Branching methods mainly : Fail
- Some solvers can not support AMPL directly : not test
- Knitro : Good result using Active Set Method

objective 192; feasibility error 1.49e-11
7 major iterations; 12 function evaluations
x0 = 1
y0 = 10
x1 = 1
y1 = 2
x2 = 7.9282
y2 = 6

#GAMS/Baron
#objective 192
#x0 = 1
#y0 = 6
#x1 = 7.9282
#y1 = 10
#x2 = 7.9282
#y2 = 2

- Please note that this example shows both objective and constraints containing integer variables

High Performance Computing in Mathematical Optimization

High Performance Computing in solvers :

interior point methods, active set methods, mixed integer branch and bound, stochastic programming and global optimization.

Three types of OpenMP parallelism

- Loop parallelism and Nested Loop parallelism
- Section parallelism
- Task parallelism

Conclusion/Discussion

- AMPL/GAMS with Knitro for initial testing
- Parallel programming using OpenMP in Fortran calling Knitro with Cplex and Intel MKL
- Cplex and Intel MKL support OpenMP
- MINLP solvers are suitable for water systems:

  Optimal Design, Flow Management :: Multiple Objective NLP

  Network Optimization :: Accurate Method

  Economic dispatch with nonsmooth cost functions :: Fast NLP

  Security constrained :: Stable convergence